Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 02 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	Area $= \frac{1}{2}\int_0^{\pi} r^2 \, \mathrm{d}\theta = \frac{1}{2}\int_0^{\pi} \left[2\sqrt{\sinh\theta + \cosh\theta}\right]^2 \{\mathrm{d}\theta\}$ or $\frac{1}{2}\int_0^{\pi} 4(\sinh\theta + \cosh\theta)\,\{\mathrm{d}\theta\}$	**B1**	1.1b
	$= 2\left[\cosh\theta + \sinh\theta\right]_0^{\pi}$ or $2\int_0^{\pi}\left(\frac{e^{\theta} - e^{-\theta}}{2} + \frac{e^{\theta} + e^{-\theta}}{2}\right)\mathrm{d}\theta = 2\int_0^{\pi} e^{\theta}\,\mathrm{d}\theta = 2\left[e^{\theta}\right]_0^{\pi}$	**M1**	1.1b
	$= 2\left((\cosh\pi + \sinh\pi) - (\cosh 0 + \sinh 0)\right)$ $= 2\left(\left(\frac{e^{\pi} + e^{-\pi}}{2} + \frac{e^{\pi} - e^{-\pi}}{2}\right) - (1+0)\right)$ or $= 2\left(e^{\pi} - e^{0}\right)$	**M1**	3.1a
	$= 2e^{\pi} - 2$ or $2\left(e^{\pi} - 1\right)$	**A1**	2.1
		(4)	
		(4 marks)	

Notes:

B1: Correct area formula applied, including the $\frac{1}{2}$ and correct limits, may be seen later, $\mathrm{d}\theta$ may be implied.

M1: Attempts the integration $\sinh\theta \rightarrow \pm\cosh\theta$ and $\cosh\theta \rightarrow \pm\sinh\theta$ or in terms of exponentials $\int e^{\lambda\theta}\,\mathrm{d}\theta = \frac{1}{\lambda}e^{\lambda\theta}$

M1: Applies their limits to the integral, subtracts (there must be an attempt to integrate) and uses exponential definitions to achieve answer in suitable form. Condone the inclusion of i or a missing ½ from the definitions. This can be implied.

A1: Correct exact answer, no i

Question	Scheme	Marks	AOs
2(a)	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	**B1**	1.1b
		(1)	
(b)	**Version 1** $e^{(e^x - 1)} = e^{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots - 1}$ **or** **Version 2** $e^{(e^x - 1)} = 1 + (e^x - 1) + \frac{(e^x - 1)^2}{2!} + \frac{(e^x - 1)^3}{3!} + \ldots$	**M1**	1.1b
	Version 1.1 $= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) + \frac{1}{2}\left(x + \frac{x^2}{2!} + \ldots\right)^2 + \frac{1}{6}(x + \ldots)^3 + \ldots$ **Or** **Version 1.2** $e^{x + \frac{x^2}{2} + \frac{x^3}{6}} = e^x \times e^{\frac{x^2}{2}} \times e^{\frac{x^3}{6}}$ $= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(1 + \frac{x^2}{2} + \ldots\right)\left(1 + \frac{x^3}{6} + \ldots\right)$ **Or** **Version 2.1** $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \ldots$ $= \frac{1}{3} + \frac{1}{6}e^{3x} + \frac{1}{2}e^x$ $= \frac{1}{3} + \frac{1}{6}\left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6}\right) + \frac{1}{2}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$ **Or** **Version 2.2** $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \ldots$	**M1**	3.1a

	$=1+\left(x+\frac{x^2}{2}+\frac{x^3}{6}\right)+\frac{1}{2}\left[\left(1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}\right)-2\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)+1\right]$ $+\frac{1}{6}\left[\left(1+3x+\frac{(3x)^2}{2}+\frac{(3x)^3}{6}\right)-3\left(1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}\right)+3\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)-1\right]$		
	$=1+x+\left(\frac{1}{2}+\frac{1}{2}\right)x^2+\left(\frac{1}{6}+\frac{1}{2}\times 2\times\frac{1}{2}+\frac{1}{6}\right)x^3+\ldots$	**dM1**	1.1b
	$=1+x+x^2+\frac{5}{6}x^3+\ldots$	**A1** **A1**	1.1b 2.1
		(5)	
			(6 marks)

Notes:

(a)

B1: Correct series (ignore terms beyond x^3).

(b)

M1: Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! and 6 for 3! in the cube term. Follow through on their series seen in (a)

M1: A complete attempt to use the exponential Maclaurin series to produce a cubic expression in terms of x only. Allow if the 3! is incorrect for this mark, but a polynomial in x must have been achieved. Condone a slip with one term. Follow through on their series seen in (a)

dM1: Dependent on previous method mark only**.** Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power).

A1: Any two correct from coefficients of x, x^2 and x^3, need not be simplified.

A1: Fully correct answer with simplified terms.

NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark.

Special case: Using $e^{(e^x-1)}=e^{e^x}\times e^{-1}$ can score M1M1M0A0A0 for using Maclaurin series e^{e^x} and then on e^x, e^{2x}, e^{3x}

Question	Scheme	Marks	AOs
3(a)	$\mathbf{M}^2+11\mathbf{M}=\begin{pmatrix}a & 0\\ 0 & a\end{pmatrix}\Rightarrow\begin{pmatrix}34 & 5k-10\\ 6k-12 & k^2+30\end{pmatrix}+\begin{pmatrix}-22 & 55\\ 66 & 11k\end{pmatrix}=\begin{pmatrix}a & 0\\ 0 & a\end{pmatrix}$	**M1**	1.1b
	$\Rightarrow a=12$	**A1**	2.2a
	$5k-10+55=0\Rightarrow 5k=-45\Rightarrow k=-9*$ or $6k-12+66=0\Rightarrow 6k=-54\Rightarrow k=-9*$ or $k^2+11k+30=12\Rightarrow k^2+11k+18=0\Rightarrow k=-2,-9*$, $k\neq-2$ as $5\times-2-10+55\neq0$ or $6\times-2-12+66\neq0$	**A1***	2.1
		(3)	
	Alternative Using $k=-9$ $\mathbf{M}^2+11\mathbf{M}=\begin{pmatrix}a & 0\\ 0 & a\end{pmatrix}\Rightarrow\begin{pmatrix}34 & -55\\ -66 & 111\end{pmatrix}+\begin{pmatrix}-22 & 55\\ 66 & -99\end{pmatrix}=\begin{pmatrix}12 & 0\\ 0 & 12\end{pmatrix}$	**M1**	1.1b
	$\Rightarrow a=12$	**A1**	2.2a
	Conclusion: therefore $k=-9$	**A1***	2.1
		(3)	
(b)	$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ mx+c\end{pmatrix}=\begin{pmatrix}X\\ mX+c\end{pmatrix}\Rightarrow\begin{cases}-2x+5(mx+c)=X\\ 6x-9(mx+c)=mX+c\end{cases}$	**M1**	1.1b
	$\Rightarrow 6x-9mx-9c=-2mx+5m^2x+5mc+c$ $\left\{\Rightarrow\left(5m^2+7m-6\right)x+\left(5m+10\right)c=0\right\}$	**M1** **A1**	3.1a 1.1b
	$\Rightarrow 5m^2+7m-6=0\left\{\Rightarrow(m+2)(5m-3)\right\}\Rightarrow m=-2,\frac{3}{5}$	**M1**	1.1b
	$m=\frac{3}{5}\Rightarrow 5m+10\neq0$ so need $c=0$ hence $y=\frac{3}{5}x$ is a fixed line	**A1**	2.2a
	$m=-2\Rightarrow 5m+10=0$ so c can be anything, so $y=-2x+c$ for any c is fixed.	**A1**	2.2a
		(6)	
(c)	($(0,c)\to(5c,-9c)$ so need c = 0,) $(1,m)\to(-2+5m,6-9m)$ so need or $5m=3$ hence $y=\frac{3}{5}x$ contains fixed points.	**B1**	3.2a

	or $$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ \frac{3}{5}x\end{pmatrix}=\begin{pmatrix}-2x+3x\\ 6x-\frac{27}{5}x\end{pmatrix}=\begin{pmatrix}x\\ \frac{3}{5}x\end{pmatrix}\Rightarrow y=\frac{3}{5}x$$ or $$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow\begin{cases}-2x+5y=x\\ 6x-9y=y\end{cases}\Rightarrow y=\frac{3}{5}x$$		
		(1)	
			(10 marks)

Notes:

(a)

M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of *a* deduced

A1*: Correct work to show $k=-9$. If off diagonals are used no further justification is needed (they are "given" the result is true). If the bottom right entry is used there must be a valid reason for rejecting -2 as a solution (ie checking the off diagonal).

Alternative: Using $k=-9$

M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of *a* deduced

A1*: Draws the conclusion that $k=-9$

(b)

M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates "*X*" to get a linear equation in "*x*".

A1: Correct equation need not be simplified isw

M1: Solves their quadratic equation in *m* by any valid means including calculator

A1: Deduces $y=\frac{3}{5}x$ is a fixed line (where c = 0). If the value for *m* here is wrong, allow this A for $y=-2x$ if the general case for the final A is not scored.

A1: Deduces $y=-2x+c$ is a fixed line where *c* can be any value. Must include all the lines.

Note: $y=\frac{3}{5}x$ and $y=-2x$ scores final A1 A0

<u>Special Case 1</u> M1M1A0M1A1A0

$$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ mx\end{pmatrix}=\begin{pmatrix}X\\ mX\end{pmatrix}\Rightarrow\begin{cases}-2x+5(mx)=X\\ 6x-9(mx)=mX\end{cases}$$

M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates "*X*" to get a linear equation in "x". $\Rightarrow 6x-9mx=-2mx+5m^2x$

A0: Incorrect equation.

M1: Solves their quadratic equation in *m* by any valid means.

$\Rightarrow 5m^2+7m-6=0\Rightarrow(m+2)(5m-3)\Rightarrow m=-2,\frac{3}{5}$

A1: Deduces $y=\frac{3}{5}x$ is a fixed line (where *c* = 0). If the value for *m* here is wrong, allow this A for $y=-2x$ if the general case for the final A is not scored.

A0: Incorrect equation

<u>Special Case 2</u> Finding the line of invariant points M1M0A0M0A1A0

$$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow\begin{cases}-2x+5y=x\\ 6x-9y=y\end{cases}$$

M1: Sets up the matrix equation for a line of invariant points and extracts the simultaneous equations from the matrix equation.

M0 A0 M0

A1: Deduces $y=\frac{3}{5}x$ is a fixed line

A0: Incorrect equation

Outside the specification

M1: Find the eigenvalues $\begin{pmatrix}-2-\lambda & 5\\ 6 & -9-\lambda\end{pmatrix}\Rightarrow(2-\lambda)(-9-\lambda)-6\times5=0$ leading to a 3TQ and solves to find a value for λ, $\lambda^2+11\lambda-12=0\Rightarrow\lambda=...\{1,-12\}$

M1: Uses one of their eigenvalues to find an equation $\begin{pmatrix}-2 & 5\\ 6 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=-12\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow...\{y=-2x\}$ or

$$\begin{pmatrix} -2 & 5 \\ 6 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow ...\{5y = 3x\}$$

A1: One correct equation

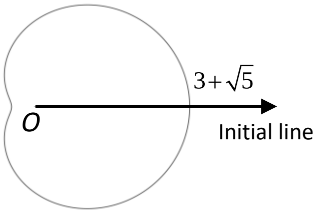
M1: Uses both of their eigenvalues to find an equation

A1: Deduces $y = \frac{3}{5}x$

A1: Deduces $y = -2x + c$

(c)

B1: Identifies $y = \frac{3}{5}x$ is a line of fixed points with reason. Allow if c = 0 is assumed. See scheme for possible reason.

Question	Scheme	Marks	AOs
4(a)	 Recalls correct shape for the type of curve, including ‘dimple’	**B1**	1.2
	Correct position with labelling of pole, initial line and point.	**B1**	1.1b
		(2)	
(b)	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(3\sin\theta+\sqrt{5}\sin\theta\cos\theta\right)=A\cos\theta+B\cos 2\theta$ $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left(3+\sqrt{5}\cos\theta\right)\sin\theta\right)=A\sin^2\theta+B\cos\theta+C\cos^2\theta$	**M1**	1.1b
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(3\sin\theta+\sqrt{5}\sin\theta\cos\theta\right)=3\cos\theta+\sqrt{5}\cos 2\theta$ $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left(3+\sqrt{5}\cos\theta\right)\sin\theta\right)=-\sqrt{5}\sin^2\theta+3\cos\theta+\sqrt{5}\cos^2\theta$	**A1**	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 3\cos\theta+\sqrt{5}\left(2\cos^2\theta-1\right)=0$ or $-\sqrt{5}\left(1-\cos^2\theta\right)+3\cos\theta+\sqrt{5}\cos^2\theta=0$ Leading to a quadratic in $\cos\theta$ $\left\{2\sqrt{5}\cos^2\theta+3\cos\theta-\sqrt{5}=0\right\}$	**M1**	3.1a
	$\cos\theta=\frac{1}{\sqrt{5}}$ following a correct quadratic, any extra solutions are rejected { $\cos\theta=\frac{-3\pm 7}{4\sqrt{5}}$, quadrant 1 needs $\cos\theta>0$ }	**A1**	2.3
		(4)	
(c)	$r=4$	**B1**	1.1b
		(1)	
			(7 marks)

Notes:

(a)

B1: Recalls the correct cardioid shape for this type of polar curve.

B1: Correctly placed with the pole, initial line and point where curve crosses the initial line all indicated in some way.

(b)

M1: Uses $y = r\sin\theta$ with the curve and attempts to differentiate. Accept any correct form but may have slips in coefficients, so e.g. as shown or $A\cos\theta + B\cos^2\theta + C\sin^2\theta$ can score M1.

A1: Correct differentiation. Accept equivalents, e.g. $3\cos\theta + \sqrt{5}\cos^2\theta - \sqrt{5}\sin^2\theta$

M1: Sets their derivative equal to zero (may be implied), using trig identities to form a quadratics for $\cos\theta$

Allow this mark form the use of $r\cos\theta$ leads to $-3\sin\theta - 2\sqrt{5}\sin\theta\cos\theta = 0$, score M1 for factorising out $\sin\theta$ and finds a value for $\cos\theta$

A1: Solves their quadratic and selects the correct value for $\cos\theta$. If the other value is given it is A0 unless clearly rejected.

(c)

B1: Correct value for *r*

Question	Scheme	Marks	AOs
5(a)	$\alpha=\frac{z_1+z_2}{2}=\frac{35-25\text{i}-29+39\text{i}}{2}=\ldots$ $\alpha=z_1+\frac{1}{2}\overrightarrow{z_1z_2}=35-25\text{i}+\frac{1}{2}(-64+64\text{i})=\ldots$ $\alpha=z_2+\frac{1}{2}\overrightarrow{z_2z_1}=-29+39+\frac{1}{2}(64-64\text{i})=\ldots$	**M1**	1.1b
	$=3+7\text{i}\,*$	**A1***	1.1b
		(2)	
(b)	$\beta(z_1-\alpha)=\left(\frac{1+\text{i}}{64}\right)(35-25\text{i}-(3+7\text{i}))=\left(\frac{1+\text{i}}{64}\right)(32-32\text{i})=$ $=\frac{1}{64}(32-32\text{i}+32\text{i}-32\text{i}^2)=\frac{1}{64}(32-32\text{i}+32\text{i}+32)$	**M1**	1.1b
	$=\frac{1}{64}(64)=1*$	**A1***	1.1b
		(2)	
(c)(i)	Roots are $\left\{\text{e}^0\left(\text{or }1\text{ or e}^{\text{i}2\pi}\right)\right\},\ \text{e}^{\text{i}\frac{\pi}{3}},\text{e}^{\text{i}\frac{2\pi}{3}},\ \text{e}^{\text{i}\pi},\ \text{e}^{\text{i}\frac{4\pi}{3}},\ \text{e}^{\text{i}\frac{5\pi}{3}}$ or $\text{e}^{\text{i}\frac{k\pi}{3}},\quad k=0,1,2,3,4,5$ $\left\{\text{e}^0\left(\text{or }1\text{ or e}^{\text{i}2\pi}\right)\right\},\ \text{e}^{\text{i}\frac{\pi}{3}},\text{e}^{\text{i}\frac{2\pi}{3}},\ \text{e}^{\text{i}\pi},\ \text{e}^{-\text{i}\frac{\pi}{3}},\ \text{e}^{-\text{i}\frac{2\pi}{3}}$ or $\text{e}^{\text{i}\frac{k\pi}{3}},\quad k=-2,-1,0,1,2,3,$	**B1**	1.1b
		(1)	
(c)(ii)	$w=\beta(z-\alpha)=\text{e}^{\text{i}\frac{k\pi}{3}}\Rightarrow z=\frac{\text{e}^{\text{i}\frac{k\pi}{3}}}{\beta}+\alpha$	**M1**	3.1a
	$\Rightarrow z=\frac{64\left(\cos\frac{k\pi}{3}+\text{i}\sin\frac{k\pi}{3}\right)(1-\text{i})}{(1+\text{i})(1-\text{i})}+3+7\text{i}=\ldots$	**M1**	1.1b
	Two of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\text{i}$ $\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\text{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\text{i}$ $\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\text{i}$ Or four correct decimal answers 46.7 + 18.7i – 40.7 – 4.7i 14.7 + 50.7i – 8.7 – 36.7i	**A1**	2.5

	All four of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$	**A1**	2.2a
		(4)	
	Alternative 1 $\left(\beta(z-\alpha)\right)^6=1^6 \Rightarrow (z-\alpha)^6=\dfrac{1}{\beta^6}=8589934459\mathrm{i}$ $r=\sqrt[6]{858993459}=32\sqrt{2}$ or $45.25...$ and $\theta=\dfrac{\pi}{12}+\dfrac{k\pi}{3}$ or $\theta=-\dfrac{\pi}{4}+\dfrac{k\pi}{3}$	**M1**	3.1a
	$z=r\left(\cos\theta-\mathrm{i}\sin\theta\right)+3+7\mathrm{i}=...$	**M1**	1.1b
	Two of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$ Or four correct decimal answers 46.7 + 18.7i – 40.7 – 4.7i 14.7 + 50.7i – 8.7 – 36.7i	**A1**	2.5
	All four of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\quad\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$	**A1**	2.2a
		(4)	
	Alternative 2 Rotation matrix $\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$ and $\begin{pmatrix}35\\-25\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ or $\begin{pmatrix}-29\\39\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ Or find the exponential form for $\begin{pmatrix}35\\-25\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ or $\begin{pmatrix}-29\\39\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ $32\sqrt{2}\mathrm{e}^{\frac{\pi}{4}}$ or $32\sqrt{2}\mathrm{e}^{-\frac{\pi}{4}}$	**M1**	3.1a
	$\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}32\\-32\end{pmatrix}+\begin{pmatrix}3\\7\end{pmatrix}=...$ or $\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}-32\\32\end{pmatrix}+\begin{pmatrix}3\\7\end{pmatrix}=...$ Or	**M1**	1.1b

	$32\sqrt{2}e^{\frac{\pi}{4}i}\times e^{\frac{\pi}{3}i}=\ldots$ then applies $r(\cos\theta-i\sin\theta)+3+7i=\ldots$ $32\sqrt{2}e^{-\frac{\pi}{4}i}\times e^{\frac{\pi}{3}i}=\ldots$ then applies $r(\cos\theta-i\sin\theta)+3+7i=\ldots$		
	Two of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)i$ $\quad\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)i$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)i$ $\quad\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)i$ Or four correct decimal answers 46.7 + 18.7i – 40.7 – 4.7i 14.7 + 50.7i – 8.7 – 36.7i Or as coordinates	**A1**	2.5
	All four of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)i$ $\quad\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)i$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)i$ $\quad\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)i$	**A1**	2.2a
		(4)	
		(11 marks)	

Notes:

(a)

M1: Attempts the midpoint of z_1 and z_2

A1*: Correct point.

(b)

M1: Substitutes into the equation with z_1 and α and β, simplifies and expands and applies $i^2=-1$, this may be implied by their working.

A1*: Completes the proof to find the correct answer with no errors seen, all necessary brackets as required

Mark (c) as one

(c) (i)

B1: Correct roots, accept all 6 listed or given in general form as in the scheme. Need not show the 1.

(c)(ii)

M1: Realises the need to set the roots of unity equal to $\beta(z-\alpha)$ and solve for *z*. Must be attempted at least once with any of their roots.

M1: Finds the Cartesian form for their equation for at least one of the roots other than z_1 and z_2

A1: At least two correct other roots than z_1 and z_2 in Cartesian form.

A1: Deduces all four correct in Cartesian form and no extra solutions

Alternative 1

M1: Finds the modulus and argument of $(z-\alpha)^6$

M1: Finds the Cartesian form for one of their modulus and arguments

A1A1: same as above

Alternative 2

M1: Finds the rotation matrix and subtracts the centre from z_1 or z_2. Or finds the exponential from for $z_1 - \alpha$ or $z_2 - \alpha$

M1: Finds the Cartesian form by multiplying by the rotation matrix and adding the centre. Or multiplies by $e^{\frac{\pi}{3}i}$ write in Cartesian form and adds on the centre

A1A1: same as above

Note all four correct decimal answers or written as coordinates score **A1A0**

46.7 + 18.7i   – 40.7 – 4.7i   14.7 + 50.7i   – 8.7 – 36.7i

Note: Correct answers implies the method marks

Question	Scheme	Marks	AOs
6	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{2x}\sinh x + \mathrm{e}^{2x}\cosh x = a\mathrm{e}^{2x}\sinh x + b\mathrm{e}^{2x}\cosh x$ or $\mathrm{e}^{2x}\left(a\sinh x + b\cosh x\right)$	**M1**	2.2a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}\left(2\sinh x + \cosh x\right)$ $n = 1$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}\left(\frac{3+1}{2}\sinh x + \frac{3-1}{2}\cosh x\right)$ {so the result is true for $n = 1$}	**A1**	2.4
	(Assume the result is true for $n = k$, then) Must be an attempt at the product rule, with k's in all terms $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = A\mathrm{e}^{2x}\left(\mathrm{f}\left(k\right)\sinh x + \mathrm{g}\left(k\right)\cosh x\right) + \mathrm{e}^{2x}\left(\mathrm{f}\left(k\right)\cosh x + \mathrm{g}\left(k\right)\sinh x\right)$ $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = 2\mathrm{e}^{2x}\left(\frac{3^k+1}{2}\sinh x + \frac{3^k-1}{2}\cosh x\right) + \mathrm{e}^{2x}\left(\frac{3^k+1}{2}\cosh x + \frac{3^k-1}{2}\sinh x\right)$	**M1**	2.1
	$=\mathrm{e}^{2x}\left(\left(3^k+1+\frac{3^k-1}{2}\right)\sinh x + \left(3^k-1+\frac{3^k+1}{2}\right)\cosh x\right)$ or $= \mathrm{e}^{2x}\left(\frac{3\times 3^k+1}{2}\sinh x + \frac{3\times 3^k-1}{2}\cosh x\right)$	**dM1**	1.1b
	$= \mathrm{e}^{2x}\left(\frac{3^{k+1}+1}{2}\sinh x + \frac{3^{k+1}-1}{2}\cosh x\right)$	**A1**	2.1
	<u>If true for $n = k$ then true for $n = k + 1$</u>, and as also <u>true for $n = 1$</u>, so the result is <u>true for all positive integers</u> or <u>true $n \in \mathbb{N}$</u>	**A1**	2.4
		(6)	
	Alternative using exponential definitions $y = \mathrm{e}^{2x}\sinh x \Rightarrow y = \mathrm{e}^{2x}\left(\frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}\right) = \frac{1}{2}\left(\mathrm{e}^{3x} - \mathrm{e}^x\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(3\mathrm{e}^{3x} - \mathrm{e}^x\right)$	**M1**	2.2a
	$n = 1$ then	**A1**	2.4

	$\frac{dy}{dx}=e^{2x}\left(\frac{3+1}{2}\sinh x+\frac{3-1}{2}\cosh x\right)=e^{2x}\left(2\frac{e^x-e^{-x}}{2}+\frac{e^x+e^{-x}}{2}\right)$ $=\frac{1}{2}\left(3e^{3x}-e^x\right)$		
	(Assume the result is true for $n = k$, then) $\frac{d^k y}{dx^k}=e^{2x}\left(\frac{3^k+1}{2}\left(\frac{e^x-e^{-x}}{2}\right)+\frac{3^k-1}{2}\left(\frac{e^x+e^{-x}}{2}\right)\right)$ $=e^{2x}\left(\frac{3^k}{2}e^x-\frac{1}{2}e^{-x}\right)=\frac{3^k}{2}e^{3x}-\frac{1}{2}e^x$ Then differentiates $\frac{d^{k+1}y}{dx^{k+1}}=e^{2x}\left(\frac{3^{k+1}+1}{2}\left(\frac{e^x-e^{-x}}{2}\right)+\frac{3^{k+1}-1}{2}\left(\frac{e^x+e^{-x}}{2}\right)\right)$	**M1**	2.1
	Simplifies $\frac{d^{k+1}y}{dx^{k+1}}=e^{2x}\left(\frac{3^{k+1}}{2}e^x-\frac{1}{2}e^{-x}\right)=\frac{3^{k+1}}{2}e^{3x}-\frac{1}{2}e^x$	**dM1**	1.1b
	Using the given result $\frac{d^{k+1}y}{dx^{k+1}}=e^{2x}\left(\frac{3^{k+1}+1}{2}\left(\frac{e^x-e^{-x}}{2}\right)+\frac{3^{k+1}-1}{2}\left(\frac{e^x+e^{-x}}{2}\right)\right)$ $=3\times\frac{3^k}{2}e^{3x}-\frac{1}{2}e^x=\frac{3^{k+1}}{2}e^{3x}-\frac{1}{2}e^x$	**A1**	2.1
	<u>If true for $n = k$ then true for $n = k + 1$</u>, and as also <u>true for $n = 1$</u>, so the result is <u>true for all positive integers</u> or <u>true $n\in$</u>□	**A1**	2.4
		(6)	
			(6 marks)

Notes:

M1: Differentiates to a correct form

A1: Correct derivative and reaches appropriate form to deduce the result is true for $n = 1$, minimum $\frac{dy}{dx}=e^{2x}\left(\frac{4}{2}\sinh x+\frac{2}{2}\cosh x\right)$

M1: (Makes the inductive assumption and) attempts the (k +1) th derivative from the k th derivative. Allow slips in coefficients but must be evidence of the use of the product rule. Must be of the form

$$\frac{d^{k+1}y}{dx^{k+1}}=Ae^{2x}\left(f(k)\sinh x+g(k)\cosh x\right)+e^{2x}\left(f(k)\cosh x+g(k)\sinh x\right)$$

dM1: Dependent on the previous method mark. Factors out the exponential and gathers the sinh x and cosh x terms. Accept either form shown or equivalent.

A1: Reaches the correct form from correct work. Must have the "k+1" showing. Depends on the previous two method marks.

A1: Makes appropriate concluding sentence covering the points indicated in scheme. Depends on all method marks having been scored. Must have reached at least the second line shown in the dM mark, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown).

Maybe seen as a narrative throughout their solution.

This mark requires all necessary brackets throughout.

Alternative: using exponentials

M1: Uses the exponential definition of sinh$x$ and differentiates to a correct form

A1: Correct derivative and uses the exponential definitions of sinh$x$ and cosh$x$ to deduce the result is true for n = 1

M1: (Makes the inductive assumption and) attempts the (k +1) th derivative from the k th derivative. Uses the exponential definition of sinh$x$ first and then differentiates to a correct form

dM1: Dependent on the previous method mark. Collects exponential terms and simplifies

A1: Reaches the k + 1 th derivative. Uses the exponential definitions of sinh$x$ and cosh$x$ in the result, simplifies and achieves the correct result

A1: Makes appropriate concluding sentence covering the points indicated in scheme. Depends on all method marks having been scored. Must have reached at least the second line shown in the dM mark, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown).

Maybe seen as a narrative throughout their solution.

This mark requires all necessary brackets throughout.

Question	Scheme	Marks	AOs
7(a)	$\dfrac{\pi}{16}\displaystyle\int_{-1.545}^{1.257}\left(6-3y^2+y\cos\left(\frac{5}{2}y\right)\right)\{\mathrm{d}y\}$ $\pi\displaystyle\int_{-1.545}^{1.257}\left(\frac{3}{8}-\frac{3}{16}y^2+\frac{1}{16}y\cos\left(\frac{5}{2}y\right)\right)\{\mathrm{d}y\}$	**B1**	1.1a
	$\displaystyle\int x^2\,\mathrm{d}y=\frac{1}{16}\int 6-3y^2+y\cos\left(\frac{5}{2}y\right)\mathrm{d}y \to Ky-Ly^3+\ldots$	**M1**	1.1b
	$\displaystyle\int y\cos\left(\frac{5}{2}y\right)\mathrm{d}y = Ay\sin\left(\frac{5}{2}y\right)+B\cos\left(\frac{5}{2}y\right)(+c)$ $\left\{\displaystyle\int y\cos\left(\frac{5}{2}y\right)\mathrm{d}y = y.\frac{2}{5}\sin\left(\frac{5}{2}y\right)-\int 1.\frac{2}{5}\sin\left(\frac{5}{2}y\right)\mathrm{d}y = \frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)(+c)\right\}$	**M1**	3.1a
	$\displaystyle\int x^2\,\mathrm{d}y=\frac{1}{16}\left(6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right)(+c)$ $\displaystyle\int x^2\,\mathrm{d}y=\frac{3}{8}y-\frac{1}{16}y^3+\frac{1}{40}y\sin\left(\frac{5}{2}y\right)+\frac{1}{100}\cos\left(\frac{5}{2}y\right)(+c)$	**A1**	1.1b
	$\displaystyle\int_{-1.545}^{1.257}x^2\,\mathrm{d}y=\frac{1}{16}\left[6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right]_{-1.545}^{1.257}$ $=\dfrac{1}{16}\left(5.3954\ldots-(-6.1101\ldots)\right)=\ldots$ $=(0.3372\ldots)-(-0.3818\ldots)=\ldots$	**M1**	3.4
	Volume = $\pi\times\dfrac{11.505\ldots}{16}=2.26$ **cm³** (2.2591159…) cso	**A1**	3.2a
		(6)	
(b)	Max volume for 100 berries (as we know volume of the largest) is 100×2.26 □ 226	**B1ft**	1.1b
	Reason e.g. not all the berries will become juice (e.g. skin, flesh, seeds may not pulp) or not all will be as big as the largest, 150 < 200 or 300 > 200 If their value • is less than 220 –so not likely to produce 200 cm³ of juice. • is between 220 and 250 – they can conclude either way • is greater than 250 – so likely to produce 200 cm³ of juice.	**B1ft**	2.2b

		(1)	
(8 marks)			

Notes:

(a)

B1: Selects the correct volume of revolution formula to use, with correct limits in evidence, could appear later in their working

M1: Attempts to integrate with the correct form for the constant and term in y^2

M1: Applies integration by parts fully on the $y\cos\left(\frac{5}{2}y\right)$ term in the correct direction. Allow slips in the coefficients, but the form must be correct.

A1: Correct integration of the x^2 equation.

M1: Applies the correct limits to their integral provided there was some attempt at integration. No need for the π for this mark. **Condone working in degrees (5.74 …) - (-5.38…), award this mark following correct integration**. If their integration is incorrect you will need to check the use of limits in radians only if they do not show the substitution.

A1: Correct volume including units. Accept awrt 2.26 **cm³**. Allow 0.719π **cm³**. Correct solution only

Note: All previous marks must have been scored to award this final accuracy mark

Special case: Use of calculator for all the integration can score a maximum of **B1M0M0A0M1A0**

(b)

B1ft: Attempts to estimate the volume of juice produced by 100 berries - look for their (a) multiplied by 100.

B1ft: Draws a suitable conclusion with reason given, see scheme

Question	Scheme		Marks	AOs
8(a)	The real axis. Horizontal line through (0, 0) Line $y = 0$ Accept on a diagram	The other possibility is that all three roots have the same real part so lie on a vertical line/perpendicular to the real axis/parallel to the imaginary axis Line $x = k$ where k is a real number Accept on a diagram	**B1** **B1**	3.1a 2.2a
			(2)	
(b)	Other roots are $\frac{3}{2}$ and $\frac{3}{2}-\frac{3}{2}\mathrm{i}$		**B1**	3.2a
			(1)	
(c)(i)	Common root must be $\frac{3}{2}$		**B1**	2.2a
			(1)	
(ii)	Sets product of roots = – 12 using their $\frac{3}{2}\times -4\times\alpha=-12$ Or $\mathrm{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z-\alpha)$		**M1**	1.1b
	Solves to find a value of the third root their $\frac{3}{2}\times -4\times\alpha=-12\Rightarrow\alpha=2$ Or $\mathrm{g}(z)=\left(z-\frac{3}{2}\right)(z\pm 4)(z-\alpha)\Rightarrow -\frac{3}{2}\times 4\times -\alpha=12\Rightarrow\alpha=2$		**M1** **A1**	3.1a 1.1b
			(3)	
(d)	$8\left\{z-\frac{3}{2}\right\}\left(z-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)\left(z-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)=8\left\{z-\frac{3}{2}\right\}\left(z^2-3z+\frac{9}{2}\right)$ Or $b=-8\left(\frac{3}{2}+\frac{3}{2}+\frac{3}{2}\mathrm{i}+\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)=\ldots\{-36\}$ $c=8\left[\left(\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)\right)+\left(\frac{3}{2}\times\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)\right)+\left(\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)\right)\right]=\ldots\{72\}$ $d=-8\left[\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)\right]=\ldots\{-54\}$		**M1**	1.1b

	$f(z)=g(z) \Rightarrow 8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)=\left(z-\frac{3}{2}\right)(z+4)(z-2)$ $\Rightarrow 8z^2-24z+36=(z+4)(z-2)$ Or Either $g(z)=\left(z-\frac{3}{2}\right)(z+4)(z-2)=...$ or $P=-\left(\frac{3}{2}-4+2\right)=...\left\{\frac{1}{2}\right\}$ and $Q=\left(\frac{3}{2}\times-4\right)+\left(\frac{3}{2}\times 2\right)+(2\times-4)=...\{-11\}$ to find g(z) and sets their f(z) = their g(z) $8z^3-36z^2+72z-54=z^3+\frac{1}{2}z^2-11z+12$	**M1**	3.1a
	$7z^2-26z+44=0 \Rightarrow z=...$ or $7z^3-\frac{73}{2}z^2+83z-66=0 \Rightarrow z=...$	**M1**	1.1b
	So solutions are $\frac{3}{2}$, $\frac{13\pm i\sqrt{139}}{7}$	**A1**	1.1b
		(4)	
(11 marks)			

Notes:

(a)

B1: One correct line described

B1: Two correct lines described

Special case: If candidate states that if the **coefficients** are **complex** then any line is possible score B1B1

(b)

B1: Interprets the conclusion from (a) in context by identifying the correct two roots.

(c)

B1: Deduces the real root is the one in common.

M1: Sets product of roots = – 12 using their $\frac{3}{2}\times-4\times\alpha=-12$. Alternatively forms an equation for g(z) using the roots

M1: Solves their equation to find the third root, condone use of 12 for this mark. Alternatively multiply their constant and sets = 12 to find the third root. Condone a sign slip for this mark

A1: Correct third root

(d)

M1: Uses their roots of f(z) to form a cubic expression for f(z), and expands to at least a linear term times a quadratic with real coefficients (which may be seen later). May just expand the complex brackets.

They must have the factor 8 for this mark

Alternative uses $b = -8(\text{sum of their roots})$ $c = 8(\text{pair sum of their roots})$ and $d = -8(\text{product of their roots})$

Note: $f(z) = 8z^3 - 36z^2 + 72z - 54$

M1: Sets their expressions equal and factorises out or cancels the common term to achieve a quadratic expression in *z*. Allow if f(*z*) is not yet expanded or factor of 8 is missing.

Alternatively finds the expression for g(*z*) by multiplying out bracket or uses P = - sum roots and Q = pair sum. Sets their f(*z*) = their g(*z*) both must be cubic

M1: Expands, gathers terms and solves the resulting quadratic. Allow this mark if the $z = \frac{3}{2}$ solution is not given.

Alternatively simplifies for form a cubic = 0 and solves using calculator to find complex roots

A1: All three correct solutions given. Note decimals $\frac{13}{7} \pm \text{i}1.68...$ is A0

Special case If the candidate just forgets the factor of 8 for f(*z*) this scores M0M1M0A0

Question	Scheme	Marks	AOs
9(a)	$\frac{d^2y}{dt^2}=0.032\frac{dx}{dt}-0.025\frac{dy}{dt}$ $\quad$ $\frac{d^2y}{dt^2}=\frac{4}{125}\frac{dx}{dt}-\frac{1}{40}\frac{dy}{dt}$ $\frac{dx}{dt}=\frac{1}{0.032}\left(0.025\frac{dy}{dt}+\frac{d^2y}{dt^2}\right)$ $\quad$ $\frac{dx}{dt}=\frac{25}{32}\frac{dy}{dt}+\frac{125}{4}\frac{d^2y}{dt^2}$	B1	1.1b
	$\frac{d^2y}{dt^2}=0.032(0.025y-0.045x+2)-0.025\frac{dy}{dt}$ $\frac{d^2y}{dt^2}=0.0008y-0.00144x+0.064-0.025\frac{dy}{dt}$ $\frac{d^2y}{dt^2}=\frac{1}{1250}y-\frac{9}{6250}x+\frac{8}{125}-\frac{1}{40}\frac{dy}{dt}$ Then substitutes for x $\frac{d^2y}{dt^2}=0.0008y-\frac{0.00144}{0.032}\left(\frac{dy}{dt}+0.025y\right)+0.064-0.025\frac{dy}{dt}$ or $\frac{d^2y}{dt^2}=\frac{1}{1250}y-\frac{9}{6250}\left(\frac{125}{4}\frac{dy}{dt}+\frac{25}{32}y\right)+\frac{8}{125}-\frac{1}{40}\frac{dy}{dt}$ or $\frac{1}{0.032}\left(\frac{d^2y}{dt^2}+0.025\frac{dy}{dt}\right)=0.025y-\frac{0.045}{0.032}\left(\frac{dy}{dt}+0.025y\right)+2$	M1	1.1b
	$\left\{\frac{d^2y}{dt^2}=-0.000325y-0.07\frac{dy}{dt}+0.064\right\}$ $\left\{\frac{d^2y}{dt^2}=-\frac{13}{40000}y-\frac{7}{100}\frac{dy}{dt}+\frac{8}{125}\right\}$ $40\,000\frac{d^2y}{dt^2}+2800\frac{dy}{dt}+13y=2560*$	A1*	2.1
		(3)	
	Alternative $\frac{d^2y}{dt^2}=0.032\frac{dx}{dt}-0.025\frac{dy}{dt}$	B1	1.1b

	$40000\left[0.032\frac{\mathrm{d}x}{\mathrm{d}t}-0.025\frac{\mathrm{d}y}{\mathrm{d}t}\right]+2800\left[0.032x-0.025y\right]+\frac{13}{0.025}\left[0.032x-\frac{\mathrm{d}y}{\mathrm{d}t}\right]$ $=1280\frac{\mathrm{d}x}{\mathrm{d}t}-1000\frac{\mathrm{d}y}{\mathrm{d}t}+89.6x-70y+16.64x-520\frac{\mathrm{d}y}{\mathrm{d}t}$ $=1280\left[0.025y-0.045x+2\right]-1520\left[0.032x-0.025y\right]$ $=A$	**M1**	1.1b
	$32y-57.6x+2560-48.64x+38y-70y+106.24=2560*$	**A1***	2.1
		(3)	
(b)	$40000m^2+2800m+13\{=0\}\Rightarrow m=...$	**M1**	3.4
	$\mathrm{CF}: y=A\mathrm{e}^{m_1t}+B\mathrm{e}^{m_2t}$	**M1**	1.1b
	$\mathrm{CF}: y=A\mathrm{e}^{\frac{-t}{200}}+B\mathrm{e}^{\frac{-13t}{200}}$ $\mathrm{CF}: y=A\mathrm{e}^{-0.005t}+B\mathrm{e}^{-0.065\mathrm{t}}$	**A1**	1.1b
	PI: Try $y=k\Rightarrow 13k=2560\Rightarrow k=...\left\{\frac{2560}{13}\right\}$	**M1**	3.4
	GS: $y=A\mathrm{e}^{\frac{-t}{200}}+B\mathrm{e}^{\frac{-13t}{200}}+\frac{2560}{13}$ $GS: y=A\mathrm{e}^{-0.005t}+B\mathrm{e}^{-0.065\mathrm{t}}+\frac{2560}{13}$	**A1ft**	1.1b
		(5)	
(c)	$t=0, y=0\Rightarrow 0=A+B+\frac{2560}{13}$	**M1**	3.4
	$t=0, y=0, x=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=0.032\times0-0.025\times0=0$ Or Used $x=\frac{1}{0.032}\left(\frac{\mathrm{d}y}{\mathrm{d}t}+0.025y\right)$ to find an equation in t	**B1**	3.4
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{A}{200}\mathrm{e}^{\frac{-t}{200}}-\frac{13B}{200}\mathrm{e}^{\frac{-13t}{200}}=0\Rightarrow-\frac{A}{200}-\frac{13B}{200}=0\Rightarrow A=-13B$ Or $x=\frac{1}{0.032}\left[-0.005A\mathrm{e}^{\frac{-t}{200}}-0.065B\mathrm{e}^{\frac{-13t}{200}}+0.025\left(A\mathrm{e}^{\frac{-t}{200}}+B\mathrm{e}^{\frac{-13t}{200}}+\frac{2560}{13}\right)\right]$	**M1**	1.1b

	$x=\frac{5}{8}A\mathrm{e}^{\frac{-t}{200}}-\frac{5}{4}B\mathrm{e}^{\frac{-13t}{200}}+\frac{2000}{13}\Rightarrow 0=\frac{5}{8}A-\frac{5}{4}B+\frac{2000}{13}$		
	$y=-\frac{640}{3}\mathrm{e}^{\frac{-t}{200}}+\frac{640}{39}\mathrm{e}^{\frac{-13t}{200}}+\frac{2560}{13}$	**A1**	1.1b
		(4)	
(d)	As $t\to\infty, \mathrm{e}^{-kt}\to 0$ for $k>0$ so $y\to\ldots$,	**M1**	1.1b
	$y\to\frac{2560}{13}\approx 196$ or 197 so the rate of administration is sufficient to reach the required level.	**A1ft**	3.2b
		(2)	
			(14 marks)

Notes:

(a)
B1: Differentiates the second equation with respect to *t* correctly. May have rearranged to make *x* the subject first. The dot notation for derivatives may be used.

M1: Uses the second equation to eliminate *x* to achieve an equation in *y*, $\frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$.

A1*: Achieves the printed answer with no errors, allow dot notation

$$40\,000\ddot{y}+2800\dot{y}+13y=2560$$

Alternative

B1: Differentiates the second equation with respect to *t* correctly. May have rearranged to make *x* the subject first. The dot notation for derivatives may be used.

M1: Substitutes in the printed differential equation uses both equations to remove all derivative, form an expression involving *x*'s and *y*'s which simplifies to a constant.

A1*: Achieves 2560 with no errors seen

(b)
M1: Uses the model to form and attempt to solve the auxiliary equation the PI (Accept a correct equation followed by two values for *m* as an attempt to solve.)

M1: Forms the complementary function correct for their roots (so if repeated or complex roots found, award for appropriate form for CF). Must be in terms of *t* only (not *x*)

A1: Correct CF

M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI

A1ft: Combines their CF (which need not be correct) with the correct PI to give y in terms of t so look for y = their CF + $\frac{2560}{13}$, accepting awrt 197

(c)

M1: Uses the initial conditions of the model to set up an equation in A and B from their general solution.

B1: Uses the initial conditions of the model to find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $t = 0$. This can be implied.

Alternatively uses $x = \frac{1}{0.032}\left(\frac{\mathrm{d}y}{\mathrm{d}t} + 0.025y\right)$ to find an equation in A and B from

M1: Differentiates their general solution, substitutes $t = 0$ and sets equal to their initial value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ to form another equation in A and B and proceed at least as far as finding A in terms of B oe.

Alternative substitutes y and $\frac{\mathrm{d}y}{\mathrm{d}t}$ into their x equation and uses $x = 0$ when $t = 0$ to find an equation in A and B

A1: Correct particular solution, accepting awrt 197

(d)

M1: Uses the limit of the exponential terms is zero to find the long term limit of the concentration

A1ft: Follow through on their constant term and draws a relevant conclusion.

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